

Introduction to Separation Logic

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$$P, Q, R ::= \dots \mid \ell \mapsto v \mid P * Q \mid P \multimap Q$$
$$e ::= \dots \mid \mathbf{new} \ e \mid e_1 \leftarrow e_2 \mid !e$$
$$v ::= \dots \mid \ell$$

- $\ell \mapsto v$ reads “ ℓ points to v ”
- $P * Q$ is **separating conjunction**, semantics: disjoint heaps
- $\mathbf{new} \ e$ evaluate e to value, store that into heap at some location, return that location
- $e_1 \leftarrow e_2$ evaluate e_1 to label, evaluate e_2 to value, store value to label
- $!e$ evaluate e to label, retrieve value at that label
- ℓ label

$$P \vdash Q$$
$$\frac{}{P * Q \vdash P}$$
$$\frac{}{P * Q \vdash Q * P}$$
$$\frac{P \vdash P' \quad Q \vdash Q'}{P * Q \vdash P' * Q'}$$
$$\frac{}{P * (Q * R) \dashv\vdash (P * Q) * R}$$
$$\frac{}{(\exists x : X.Px) * Q \vdash \exists x : X.Px * Q}$$
$$\frac{}{\ell \mapsto v * \ell \mapsto w \vdash \perp}$$
$$\frac{}{\ell \mapsto v \wedge \ell \mapsto w \vdash v = w}$$

affine, not linear, because convenient